

EXISTENCE RESULTS FOR SEMILINEAR NEUTRAL FUNCTIONAL DIFFERENTIAL INCLUSIONS WITH NONLOCAL CONDITIONS

S. K. NTOUYAS AND D. O'REGAN

Abstract. In this paper we prove existence results for first and second order semilinear neutral functional differential inclusions with finite or infinite delay in Banach spaces and nonlocal conditions, via a Nonlinear Alternative for condensing maps. Our theory makes use of analytic semigroups and fractional powers of closed operators, integrated semigroups and cosine families.

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REFERENCES

- [1] M. ADIMY, H. BOUZAHIR AND K. EZZINBI, *Existence and stability for some partial functional differential equations with infinite delay*, J. Math. Anal. Appl. **294** (2004), 438-461.
- [2] W. ARENDT, *Vector valued Laplace transforms and Cauchy problems*, Israel J. Math., **59** (1987), 327-352.
- [3] M. BENCHOHRA AND S. K. NTOUYAS, *Neutral functional differential and integrodifferential inclusions in Banach spaces*, Boll. Unione Mat. Ital. Sez. B Artic. Ric. Mat., **4**, 8 (2001), 767-782.
- [4] A. BRESSAN AND G. COLOMBO, *Extensions and selections of maps with decomposable values*, Studia Math., **90** (1988), 69-86.
- [5] L. BYSZEWSKI, *Theorems about the existence and uniqueness of solutions of a semilinear evolution nonlocal Cauchy problem*, J. Math. Anal. Appl., **162** (1991), 494-505.
- [6] G. DA PRATO AND E. SINISTRARI, *Differential operators with non-dense domains*, Ann. Scuola. Norm. Sup. Pisa Sci., **14** (1987), 285-344.
- [7] H. O. FATTORINI, *Second Order Linear Differential Equations in Banach Spaces*, North-Holland Mathematics Studies, Vol. 108, North-Holland, Amsterdam, 1985.
- [8] J. A. GOLDSTEIN, *Semigroups of Linear Operators and Applications*, Oxford Univ. Press, New York, 1985.
- [9] L. GORNIOWICZ, S. K. NTOUYAS AND D. O'REGAN, *Existence and controllability results for first and second order functional semilinear differential inclusions with nonlocal conditions*, Numer. Funct. Anal. Optim., **28** (2007), 53-82.
- [10] A. GRANAS AND J. DUGUNDJI, *Fixed Point Theory*, Springer-Verlag, New York, 2003.
- [11] J.K. HALE, J.KATO, *Phase space for retarded equations with infinite delay*, Funkcial. Ekvac., **21** (1978), 11-41.
- [12] S. HEIKKILÄ AND V. LAKSHMIKANTHAM, *Monotone Iterative Techniques for Discontinuous Non-linear Differential Equations*, Marcel Dekker, New York, 1994.
- [13] E. HERNANDEZ, *Existence results for partial neutral functional differential equations with nonlocal conditions*, Dynam. Systems Appl., **11** (2002), 241-252.
- [14] Y. HINO, S. MURAKAMI AND T. NAITO, *Functional Differential Equations with Infinite Delay*, Lect. Notes in Math., **1473**, Springer-Verlag, Berlin, 1991.
- [15] H. KELLERMAN AND M. HIEBER, *INTEGRATED SEMIGROUPS*, J. Funct. Anal., **84** (1989), 160-180.
- [16] V. LAKSHMIKANTHAM AND S. LEELA, *Differential and Integral Inequalities*, vol. I, Academic Press, New York, 1969.

- [17] S. K. NTOUYAS AND D. O'REGAN, *Existence and controllability results for semilinear neutral functional differential inclusions with nonlocal conditions*, *Dissc. Math. Differ. Incl.*, **27** (2007), 213-264.
- [18] A. PAZY, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer-Verlag, New York, 1983.
- [19] W.V. PETRYSHYN AND P.M. FITZPATRICK, *A degree theory, fixed point theorems, and mapping theorems for multivalued noncompact maps*, *Trans. Amer. Math. Soc.*, **194** (1974), 1-25.
- [20] H. R. THIEME, *Integrated semigroups and integrated solutions to abstract Cauchy problems*, *J. Math. Anal. Appl.*, **152** (1990), 416-447.
- [21] C. TRAVIS AND G. WEBB, *Cosine families and abstract nonlinear second order differential equations*, *Acta Math. Hungar.* **32** (1978), 75-96.
- [22] C. TRAVIS AND G. WEBB, *An abstract second order semilinear Volterra integrodifferential equation*, *SIAM J. Math. Anal.*, **10** (1979), 412-424.