

## A VARIATIONAL APPROACH FOR ALMOST PERIODIC SOLUTIONS IN RETARDED FUNCTIONAL DIFFERENTIAL EQUATIONS

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**Abstract.** To study the a.p. (almost periodic) solutions of retarded functional differential equations in the form  $u''(t) = \int_{-r}^0 D_1 f(u(t), u(t+\theta))d\theta + \int_{-r}^0 D_2 f(u(t-\theta), u(t))d\theta + e(t)$ , we introduce variational formalisms to characterize the a.p. solutions as a critical points of functionals defined on Banach spaces of a.p. functions. We obtain an existence result of weak a.p. solutions and a result of density of the a.p. forcing terms  $e(\cdot)$  for which the equation possesses usual a.p. solutions.

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