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TWO–POINT OSCILLATIONS IN SECOND–ORDER LINEAR DIFFERENTIAL EQUATIONS

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Abstract. A second-order linear differential equation (P): $y'' + f(x)y = 0$, $x \in I$, where $I = (0,1)$ and $f \in C(I)$, is said to be two-point oscillatory on I, if all its nontrivial solutions $y \in C(\overline{I}) \cap C^2(I)$, oscillate both at $x = 0$ and $x = 1$, i.e. having sequences of infinite zeros converging to $x = 0$ and $x = 1$. It necessarily implies that all solutions $y(x)$ of (P) must satisfy the Dirichlet boundary conditions and that $f(x)$ must be singular at both end points of $\overline{I}$. We first describe a class of two-point oscillatory equations of (P). Secondly, we prove that (P) is two-point oscillatory if $f(x)$ satisfies certain Hartman-Wintner type asymptotic conditions. Furthermore, we study the arclength of the graph $G(y)$ of solutions curve $y(x)$ on I. Two-point oscillatory equation (P) is said to be two-point rectifiable (unrectifiable) oscillatory if the arclengths of all solutions are finite (infinite). We give conditions on $f(x)$ which imply (P) is two-point rectifiable (unrectifiable) oscillatory. When (P) is two-point unrectifiable oscillatory, we determine the fractal dimension of its solution curves for a special class of $f(x)$ similar to the Euler type equations when $f(x)$ is only singular at one end point of I. Finally, the preceding results motivate a study on two-sided oscillations of (P) at an interior point of $\overline{I}$.

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