

## TWO-POINT OSCILLATIONS IN SECOND-ORDER LINEAR DIFFERENTIAL EQUATIONS

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*Abstract.* A second-order linear differential equation  $(P): y'' + f(x)y = 0$ ,  $x \in I$ , where  $I = (0, 1)$  and  $f \in C(I)$ , is said to be two-point oscillatory on  $I$ , if all its nontrivial solutions  $y \in C(\bar{I}) \cap C^2(I)$ , oscillate both at  $x = 0$  and  $x = 1$ , i.e. having sequences of infinite zeros converging to  $x = 0$  and  $x = 1$ . It necessarily implies that all solutions  $y(x)$  of  $(P)$  must satisfy the Dirichlet boundary conditions and that  $f(x)$  must be singular at both end points of  $\bar{I}$ . We first describe a class of two-point oscillatory equations of  $(P)$ . Secondly, we prove that  $(P)$  is two-point oscillatory if  $f(x)$  satisfies certain Hartman-Wintner type asymptotic conditions. Furthermore, we study the arclength of the graph  $G(y)$  of solutions curve  $y(x)$  on  $I$ . Two-point oscillatory equation  $(P)$  is said to be two-point rectifiable (unrectifiable) oscillatory if the arclengths of all solutions are finite (infinite). We give conditions on  $f(x)$  which imply  $(P)$  is two-point rectifiable (unrectifiable) oscillatory. When  $(P)$  is two-point unrectifiable oscillatory, we determine the fractal dimension of its solution curves for a special class of  $f(x)$  similar to the Euler type equations when  $f(x)$  is only singular at one end point of  $I$ . Finally, the preceding results motivate a study on two-sided oscillations of  $(P)$  at an interior point of  $\bar{I}$ .

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