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A PARABOLIC REGULARIZATION PROPERTY OF p–LOGARITHMIC SOBOLEV GENERATORS

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Abstract. Let N be a Riemannian manifold, $M \subset N$ be a domain with smooth boundary, μ a positive measure on M such that M has unit μ–volume. Consider the evolution driven by the p–Laplace–type operator ($p > 2$) associated to the natural p–energy functional $\mathscr{E}^{(p)}$ constructed from μ, homogeneous Dirichlet boundary conditions on ∂M being assumed. Assume that a single suitable logarithmic inequality holds for $\mathscr{E}^{(p)}$.Then we show that the evolution brings any data belonging to the natural domain of the evolution instantaneously into L^q for any $q > 2$, with quantitative bounds on the L^q norms.

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