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CONTRACTION IN L^1 FOR A SYSTEM ARISING IN CHEMICAL REACTIONS AND MOLECULAR MOTORS

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Abstract. We prove a contraction in L^1 property for the solutions of a nonlinear reaction–diffusion system whose special cases include a system related to intracellular transport as well as reversible chemical reactions. We then consider the special case of the linear molecular motor problem and prove the existence and uniqueness of the stationary solution up to a multiplicative constant, extending to arbitrary space dimension results which were already known in the one dimensional case; this in turn implies the convergence to stationary solutions of the solutions of the time evolution linear molecular motor problem.

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