GENERALIZED SOLUTIONS TO A
NON–LIPSCHITZ GOURSAT PROBLEM

VICTOR DÉVOUÉ

Abstract. We study the semilinear wave equation in canonical form with non-Lipschitz non-linearity by using the recent theories of generalized functions. We investigate solutions to the Goursat problem. We turn this non-Lipschitz Goursat problem with irregular data into a biparameter family of problems. The first parameter replaces the problem by a family of Lipschitz problems and the second one regularizes the data. Finally the family of problems is solved in an appropriate biparametric $(\mathscr{C}, \mathscr{E}, \mathscr{P})$ algebra.

REFERENCES

[1] J.-F. COLOMBEAU, *Elementary introduction to new generalized functions*, North Holland Math. Studies 113. North-Holland, Amsterdam, 1984.

[2] G. M. CONSTANTINE, H. SAVITS, *A multivariate Faà di Bruno formula with applications,* Trans. Amer. Math. Soc., **348**, 2 (1996), 503–520.

[3] A. DELCROIX, *Remarks on the embedding of spaces of distributions into spaces of Colombeau generalized functions*, Novisad J. math., **35**, 2 (2005), 27–40.

[4] A. DELCROIX, *Some properties of $(\mathscr{C}, \mathscr{E}, \mathscr{P})$ -algebras : Overgeneration and 0th -order estimates*. Preprint.

[5] A. DELCROIX, D. SCARPALÉZOS, *Topology on Asymptotic Algebras of Generalized Functions and Applications*, Monatsh. Math., **129** (2000), 1–14.

[6] A. DELCROIX, V. DÉVOUÉ, J.-A. MARTI, *Generalized solutions of singular differential problems. Relationship with classical solutions.* J. Math. Anal. Appl., **353**, 1 (2009), 386–402.

[7] A. DELCROIX, V. DÉVOUÉ, J.-A. MARTI, *Wellposed differential problems in Algebras of Generalized Functions*, Preprint.

[8] V. DÉVOUÉ, *Sur les singularités de certains problèmes différentiels*, Thèse de doctorat, Université des Antilles et de la Guyane (2005).

[9] V. DÉVOUÉ, *On generalized solutions to the wave equation in canonical form*, Dissertationes Math., **443**, (2007), 1–69.

[10] V. DÉVOUÉ, *Generalized solutions to a non Lipschitz Cauchy problem*, J. Appl. Anal. Vol., **15**, 1 (2009), 1–32.

[11] M. GROSSER, M. KUNZINGER, M. OBERGUGGENBERGER, R. STEINBAUER, *Geometric theory of generalized functions with applications to general relativity*, Mathematics and its Applications 537. Kluwer Academic Publ., Dordrecht, 2001.

[12] J.-A. MARTI, *Fundamental structures and asymptotic microlocalization in sheaves of generalized functions*, Integral Transforms Spec. Funct., **6**, 1-4 (1998), 223–228.

[13] J.-A. MARTI, $(\mathscr{C}, \mathscr{E}, \mathscr{P})$ -*Sheaf Structures and Applications*, In: Nonlinear Theory of Generalized Functions, M. Grosser and alii, Eds. Research notes in Mathematics. Chapman & Hall/CRC (1999), 175–186.

[14] J.-A. MARTI, *Multiparametric Algebras and Characteristic Cauchy Problem*, Non-linear algebraic analysis and applications, Proceeding of the International Conference on Generalized functions (ICGF 2000). CSP (2004), 181–192.

[15] J.-A. MARTI, *Non linear Algebraic Analysis of Delta Shock wave solutions to Burger's Equation*, Pacific J. Math., **210**, 1 (2003), 165–187.

[16] J.-A. MARTI, S. P. NUIRO, V. S. VALMORIN, *A non linear Goursat problem with irregular data*, Integral Transforms Spec. Funct., **6**, 1-4 (1998), 229–246.

[17] M. NEDELJKOV, M. OBERGUGGENBERGER, S. PILIPOVIĆ, *Generalized solution to a semilinear wave equation*, Nonlinear Anal., **61** (2005), 461–475.

[18] M. OBERGUGGENBERGER, *Multiplication of Distributions and Applications to Partial Differential Equations*, Pitman Research Notes in Mathematics 259. Longman, Harlow, 1992.

[19] J. SHATAH, M. STRUWE, *Geometric Wave Equations*, CIMS Lecture Notes, vol. 2, Courant Inst. Math. Sci., New York, 1998.

[20] M. E. TAYLOR, *Partial Differential equations III. Nonlinear Equations*, Applied Mathematical Sciences 117. Springer-Verlag, New York, 1996.

Differential Equations & Applications
www.ele-math.com
dea@ele-math.com