

CHEBYSHEV'S DIFFERENTIAL EQUATION AND ITS HYERS–ULAM STABILITY

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Abstract. We solve the inhomogeneous Chebyshev's differential equation and apply this result to obtain a partial solution to the Hyers-Ulam stability problem for the Chebyshev's differential equation.

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