

## A NONOSCILLATION THEOREM FOR HALF-LINEAR DIFFERENTIAL EQUATIONS WITH DELAY NONLINEAR PERTURBATIONS

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*Abstract.* This paper deals with the oscillation problem for nonlinear differential equations with delay. A sufficient condition is obtained for the equation to have a nonoscillatory solution. The main result is the best possible in a certain sense. Examples are given to illustrate the main result.

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