OPTIMIZATION OF THE FIRST EIGENVALUE IN
PROBLEMS INVOLVING THE BI–LAPLACIAN

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Abstract. This paper concerns minimization and maximization of the first eigenvalue in problems involving the bi-Laplacian under Dirichlet boundary conditions. Physically, in case of $N = 2$, our equation models the vibration of a non homogeneous plate Ω which is clamped along the boundary. Given several materials (with different densities) of total extension $|\Omega|$, we investigate the location of these materials throughout Ω so to minimize or maximize the first eigenvalue in the vibration of the clamped plate.

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