

EXISTENCE, NONEXISTENCE AND MULTIPLICITY RESULTS FOR SEMILINEAR ELLIPTIC PROBLEMS WITH MEASURE DATA AND ABSORPTION–REACTION TERM

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Abstract. In the case where $g(u)$ appears as an absorption term, then under some additional hypotheses on g we prove that the main problem has a solution for all $\lambda > 0$ and for all positive $\mu \in L^1(\Omega)$. In the case where g appears as a reaction term, then we prove that the main problem has at least two positive solutions under suitable hypotheses on μ . The asymptotic linear case is also studied.

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