

BLOW-UP PROPERTIES FOR PARABOLIC SYSTEMS WITH LOCALIZED NONLINEAR SOURCE

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Abstract. This paper deals with blow-up properties of solutions to a semilinear parabolic system with nonlinear localized source involved a product with local terms

$$u_t = \Delta u + \exp\{\mu u(x, t) + \nu v(x_0, t)\}, \quad v_t = \Delta v + \exp\{p u(x_0, t) + q v(x, t)\}$$

with homogeneous Dirichlet boundary conditions. We investigate the influence of localized sources and local terms on blow-up properties for this system, and prove that: (i) when $m, q \leq 0$ this system possesses uniform blow-up profiles, in other words, the localized terms play a leading role in the blow-up profile for this case; (ii) when $m, q > 0$, this system presents single point blow-up patterns, or say that local terms dominate localized terms in the blow-up profile. Moreover, the blow-up rate estimates in time and space are obtained, respectively.

Mathematics subject classification (2000): 35B40, 35K65, 35K15.

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