

## EXISTENCE RESULTS FOR A SECOND ORDER IMPULSIVE FUNCTIONAL DIFFERENTIAL EQUATION WITH STATE-DEPENDENT DELAY

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*Abstract.* In this paper, we study existence of mild solutions for a second order impulsive neutral functional differential equations with state-dependent delay. By using a fixed point theorem for condensing maps combined with theories of a strongly continuous cosine family of bounded linear operators, we prove the main existence theorems. As applications of these obtained results, some practical consequences are derived for the sub-linear growth cases. And an example is also given to illustrate our main results.

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