

## SHARP ESTIMATES OF BOUNDED SOLUTIONS TO A SECOND-ORDER FORCED EQUATION WITH STRUCTURAL DAMPING

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*Abstract.* By using differential inequalities, an essentially optimal  $L^\infty(\mathbb{R}, D(A))$  bound of the unique bounded solution of  $u'' + cAu' + A^2u = f(t)$  is obtained whenever  $A = A^* \geq \lambda I$  is a bounded or unbounded linear operator on a real Hilbert space  $H$  and  $\lambda, c$  are positive constants, while  $f \in L^\infty(\mathbb{R}, H)$ .

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