

GROUND STATES OF THE QUASILINEAR PROBLEMS INVOLVING THE CRITICAL SOBOLEV EXPONENT AND POTENTIALS

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Abstract. In this paper we study a quasilinear elliptic problem which involves the critical Sobolev exponent and multiple Hardy-type terms. By the analytic technics and variational methods, the existence and nonexistence of the ground states for the problem is established under certain assumptions.

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