

ON AN EIGENVALUE PROBLEM INVOLVING THE $p(x)$ -LAPLACE OPERATOR PLUS A NON-LOCAL TERM

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Abstract. We study an eigenvalue problem involving variable exponent growth conditions and a non-local term, on a bounded domain $\Omega \subset \mathbb{R}^N$. Using adequate variational techniques, mainly based on the mountain-pass theorem of A. Ambrosetti and P. H. Rabinowitz, we prove the existence of a continuous family of eigenvalues lying in a neighborhood at the right of the origin.

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