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THE DIRICHLET PROBLEM FOR SECOND ORDER SEMILINEAR ELLIPTIC AND PARABOLIC EQUATIONS

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Abstract. In the present paper the Dirichlet problem for semilinear elliptic and parabolic equations in general form is considered. New condition guaranteeing the global solvability of this problem for a wide class of superlinear sources, including e^u and $|u|^{p-1}u$, $p>1$, is formulated. For sublinear case (for example $\ln(1+|u|)$ or $|u|^{p-1}u$, $p<1$) this condition is automatically fulfilled. Our approach gives new a priori estimate of the solution for superlinear, sublinear and linear case as well.

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