

A GENERALIZATION OF THE KALMAN RANK CONDITION FOR TIME-DEPENDENT COUPLED LINEAR PARABOLIC SYSTEMS

F. AMMAR KHODJA, A. BENABDALLAH, C. DUPAIX AND
M. GONZÁLEZ-BURGOS

Abstract. In this paper we present a generalization of the Kalman rank condition for linear ordinary differential systems to the case of systems of n coupled parabolic equations (posed in the time interval $(0, T)$ with $T > 0$) where the coupling matrices A and B depend on the time variable t . To be precise, we will prove that the Kalman rank condition $\text{rank}[A|B](t_0) = n$, with $t_0 \in [0, T]$, is a sufficient condition (but not necessary) for obtaining the exact controllability to the trajectories of the considered parabolic system. In the case of analytic matrices A and B (and, in particular, constant matrices), we will see that the Kalman rank condition characterizes the controllability properties of the system. When the matrices A and B are constant and condition $\text{rank}[A|B] = n$ holds, we will be able to state a Carleman inequality for the corresponding adjoint problem.

Mathematics subject classification (2000): 93B05, 93B07, 35K05, 35K55, 35R30.

Keywords and phrases: Kalman condition, control, observability, Carleman estimates, parabolic systems.

REFERENCES

- [1] F. AMMAR KHODJA, A. BENABDALLAH, C. DUPAIX, I. KOSTIN, *Controllability to the trajectories of phase-field models by one control force*, SIAM J. Control Optim., **42**, 5 (2003), 1661–1680.
- [2] F. AMMAR KHODJA, A. BENABDALLAH, C. DUPAIX, *Null-controllability of some reaction-diffusion systems with one control force*, J. Math. Anal. Appl., **320**, 2 (2006), 928–943.
- [3] F. AMMAR KHODJA, A. BENABDALLAH, C. DUPAIX, M. GONZÁLEZ-BURGOS, *Controllability for a class of reaction-diffusion systems: generalized Kalman's condition*, C. R. Math. Acad. Sci. Paris, **345**, 10 (2007), 543–548.
- [4] F. AMMAR KHODJA, A. BENABDALLAH, C. DUPAIX, M. GONZÁLEZ-BURGOS, *A Kalman rank condition for the localized distributed controllability of a class of linear parabolic systems*, submitted to Comm. PDE.
- [5] O. BODART, M. GONZÁLEZ-BURGOS, R. PÉREZ-GARCÍA, *Existence of insensitizing controls for a semilinear heat equation with a superlinear nonlinearity*, Comm. PDE, **29**, 7–8 (2004), 1017–1050.
- [6] J.M. CORON, *Control and Nonlinearity*, Mathematical Surveys and Monographs, 136, American Mathematical Society, Providence, RI, 2007.
- [7] E. FERNÁNDEZ-CARA, M. GONZÁLEZ-BURGOS, L. DE TERESA, *About boundary controllability of cascade heat equations*, in preparation.
- [8] E. FERNÁNDEZ-CARA, E. ZUAZUA, *The cost of approximate controllability for heat equations: The linear case*, Adv. Differential Equations, **5** (2000), 465–514.
- [9] A. FURSIKOV, O. YU. IMANUVILOV, *Controllability of evolution equations*, Lecture Notes Ser. 34, Seoul National University, Korea, 1996.
- [10] J.-M. GHIDAGLIA, *Some backward uniqueness results*, Nonlinear Anal., **10**, 8 (1986), 777–790.
- [11] M. GONZÁLEZ-BURGOS, R. PÉREZ-GARCÍA, *Controllability results for some nonlinear coupled parabolic systems by one control force*, Asymptot. Anal., **46**, 2 (2006), 123–162.
- [12] M. GONZÁLEZ-BURGOS, L. DE TERESA, *Controllability results for cascade systems of m coupled parabolic PDEs by one control force*, submitted.

- [13] S. GUERRERO, *Null controllability of some systems of two parabolic equations with one control force*, SIAM J. Control Optim., **46**, 2 (2007), 379–394.
- [14] O. YU. IMANUVILOV, M. YAMAMOTO, *Carleman inequalities for parabolic equations in Sobolev spaces of negative order and exact controllability for semilinear parabolic equations*, Publ. Res. Inst. Math. Sci., **39**, 2 (2003), 227–274.
- [15] R. E. KALMAN, P. L. FALB, M. A. ARBIB, *Topics in mathematical control theory*, McGraw-Hill Book Co., New York-Toronto, Ont.-London 1969.
- [16] H. LEIVA, *Controllability of a system of parabolic equations with non-diagonal diffusion matrix*. IMA J. Math. Control Inform., **22**, 2 (2005), 187–199.
- [17] L. M. SILVERMAN, H. E. MEADOWS, *Controllability and observability in time-variable linear systems*, SIAM J. Control, **5**, 1 (1967), 64–73.
- [18] L. DE TERESA, *Insensitizing controls for a semilinear heat equation*, Comm. PDE, **25**, 1–2 (2000), 39–72.