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VARIATIONAL PROBLEMS WITH POINTWISE CONSTRAINTS AND DEGENERATION IN VARIABLE DOMAINS

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Abstract. In this article we deal with a sequence of functionals defined on weighted Sobolev spaces. The spaces are associated with a sequence of domains Ω_s contained in a bounded domain Ω of $\mathbb{R}^n$. The main structural components of the functionals are integral functionals whose integrands satisfy a growth and coercivity condition with a weight and additional terms $\psi_s \in L^1(\Omega_s)$. For the given functionals we consider variational problems with sets of constraints for functions v of the kind $h(x,v(x)) \leqslant 0$ a.e. in Ω_s, where $h : \Omega \times \mathbb{R} \to \mathbb{R}$. We establish conditions on h and ψ_s and on the given domains, weighted spaces and functionals under which solutions of the variational problems under consideration converge in a certain sense to a solution of a limit variational problem with the set of constraints defined by the same function h.

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