

## PSEUDO-ALMOST AUTOMORPHIC SOLUTIONS TO SOME CLASSES OF NONAUTONOMOUS PARTIAL EVOLUTION EQUATIONS

TOKA DIAGANA

*Abstract.* In this paper we obtain the existence of pseudo-almost automorphic solutions to some classes of nonautonomous partial evolution equations. To illustrate our main result, we study the existence of a pseudo-almost automorphic solution to a nonautonomous heat equation.

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