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REGULARITY FOR THE NAVIER–STOKES–FOURIER SYSTEM

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Abstract. We prove the existence of strong 2-dimensional solutions for two Cauchy-Dirichlet problems to the Navier-Stokes-Fourier system which characterizes the Newtonian fluids under heat-conducting effects. The nonstationary Navier-Stokes system for an incompressible homogeneous fluid with temperature dependent viscosity is completed by the equation of balance of energy which includes the term of dissipative heating. The regularity of solutions to the problems under study is proved through compactness methods and fixed point arguments, instead assuming the existence of weak solutions to the problems.

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