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COMPUTING THE LOCATION AND THE DIRECTION OF BIFURCATION FOR SIGN CHANGING SOLUTIONS

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Abstract. We consider sign-changing solutions of the Dirichlet problem

$$u'' + \lambda f(u) = 0, \ \ 0 < x < 1, \ \ u(0) = u(1) = 0,$$

with $n \geqslant 0$ interior roots. We give a necessary and sufficient condition that a turn occurs at the solution $(\lambda, u(x))$, depending only on the maximum value of the solution $u(x)$. If a turn does occur, we give another formula allowing to compute the direction of the turn. Our results generalize those in P. Korman, Y. Li and T. Ouyang [6], where positive solutions were considered. We give similar results for Neumann problem.

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