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# BOUNDARY BEHAVIOR FOR SOLUTIONS OF SINGULAR QUASI–LINEAR ELLIPTIC EQUATIONS

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*Abstract.* In this paper, for $1 \leqslant \gamma \leqslant 3$ our main purpose is to consider the quasilinear elliptic equation: $\operatorname{div}(|\nabla u|^{m-2}\nabla u) + (m-1)u^{-\gamma} = 0$ on a bounded smooth domain $\Omega \subset \mathbb{R}^N$, $N > 1$. We get some first-order estimates of a nonnegative solution $u$ satisfying $u = 0$ on $\partial\Omega$. For $\gamma = 1$, we find the estimate: $\lim_{x\to\partial\Omega} u(x)/p(\delta(x)) = 1$, where $p(r) \approx r\sqrt[m]{m\log(1/r)}$ near $r = 0$, $\delta(x)$ denotes the distance from $x$ to $\partial\Omega$. For $1 < \gamma \leqslant 3$, we obtain

$$\lim_{x\to\partial\Omega} \frac{u(x)}{(b_\gamma \delta(x))^{\frac{m}{\gamma+(m-1)}}} = 1,$$

where $b_\gamma = \frac{\gamma+(m-1)}{m}\left(\frac{m}{\gamma-1}\right)^{\frac{1}{m}}$.

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