

COMPARISON, EXISTENCE AND REGULARITY RESULTS FOR A CLASS OF NON-UNIFORMLY ELLIPTIC EQUATIONS

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Abstract. We prove comparison, existence and regularity results for problems whose model case is:

$$\begin{cases} -\operatorname{div} \left(\frac{Du}{(1+|u|)^\theta} \right) + \lambda u = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded open set in \mathbb{R}^N , $N > 2$, $\theta \geq 0$ and $\lambda > 0$.

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