

ASYMPTOTIC BEHAVIOUR OF SOLUTIONS OF A SECOND ORDER NEUTRAL EQUATION OF DISCRETE TYPE WITH OSCILLATING COEFFICIENTS

RADHANATH RATH AND B. L. SIBABRATA BARIK

Abstract. In this paper, we obtain sufficient conditions so that every solution of neutral functional difference equation

$$\Delta^2(y_n - p_n y_{\tau(n)}) + q_n G(y_{\sigma(n)}) = f_n,$$

oscillates or tends to zero as $n \rightarrow \infty$, where the sequence $\{q_n\}$ may change sign. Here Δ is the forward difference operator given by $\Delta x_n = x_{n+1} - x_n$, $\{\tau_n\}$ and $\{\sigma_n\}$ are increasing sequences, which are less than n and approaches ∞ as n approaches ∞ . This paper generalizes and extends some recent results.

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