

ON DEGENERATE NON-UNIFORMLY ELLIPTIC PROBLEMS

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Abstract. We are interested in the degenerate problem: $b(v) - \operatorname{div}A(v, \nabla g(v)) = f$ in Ω with the boundary condition $v = a$, where $a : \partial\Omega \rightarrow \mathbb{R}$ is measurable such that $g(a) = 0$. We suppose that the vector field A satisfies the Leray-Lions conditions, that b, g are continuous, nondecreasing with $\lim_{r \rightarrow \pm\infty} |b + g|(r) < +\infty$, that g has a flat region $[A_1, A_2]$ and is strictly increasing on $\mathbb{R} \setminus [A_1, A_2]$ for some $A_1 \leq 0 \leq A_2$. Using monotonicity methods, we prove the existence and uniqueness of a renormalized entropy solution (with possibly infinite values).

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