

## ON DEGENERATE NON-UNIFORMLY ELLIPTIC PROBLEMS

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**Abstract.** We are interested in the degenerate problem:  $b(v) - \operatorname{div}A(v, \nabla g(v)) = f$  in  $\Omega$  with the boundary condition  $v = a$ , where  $a : \partial\Omega \rightarrow \mathbb{R}$  is measurable such that  $g(a) = 0$ . We suppose that the vector field  $A$  satisfies the Leray-Lions conditions, that  $b, g$  are continuous, nondecreasing with  $\lim_{r \rightarrow \pm\infty} |b + g|(r) < +\infty$ , that  $g$  has a flat region  $[A_1, A_2]$  and is strictly increasing on  $\mathbb{R} \setminus [A_1, A_2]$  for some  $A_1 \leq 0 \leq A_2$ . Using monotonicity methods, we prove the existence and uniqueness of a renormalized entropy solution (with possibly infinite values).

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