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LOSS OF REGULARITY OF WEAK SOLUTIONS OF p–LAPLACE EQUATIONS FOR $p \neq 2$

DARKO ŽUBRINIĆ

Abstract. If $1 < p < \infty$ and $p \neq 2$ then the exponent $\gamma_c = p/|p-2|$ is critical for the pointwise loss of regularity of the p-Laplace equation $-\Delta_p u = F(x)$, $u \in W_0^{1,p}(\Omega)$, where Ω is a bounded domain in $\mathbb{R}^N$, and $F \in L^{p'}(\Omega)$. By this we mean the following: if $1 < p < 2$ and N is large enough, and the right-hand side F has a singularity of order $\gamma > \gamma_c$ at some point $a \in \Omega$, that is, $F(x) \simeq |x-a|^{-\gamma}$ in a neighbourhood of a, then at the same point the weak solution u has singularity of order which is larger than γ. The value of γ_c is optimal. For $p > 2$ we have the loss of regularity in the sense that if $F(x) = C|x|^m$ with $m > 0$, then $u(x) = u(0) + D|x|^\mu$ with $\mu < m$, provided $m > \gamma_c$. We show that the p-Laplace operator is not hypoelliptic for $p \in (1,\infty) \setminus \{1 + 1/n : n \in 2\mathbb{N} - 1\}$.

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