

LOSS OF REGULARITY OF WEAK SOLUTIONS OF p -LAPLACE EQUATIONS FOR $p \neq 2$

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Abstract. If $1 < p < \infty$ and $p \neq 2$ then the exponent $\gamma_c = p/|p-2|$ is critical for the pointwise loss of regularity of the p -Laplace equation $-\Delta_p u = F(x)$, $u \in W_0^{1,p}(\Omega)$, where Ω is a bounded domain in \mathbb{R}^N , and $F \in L^p(\Omega)$. By this we mean the following: if $1 < p < 2$ and N is large enough, and the right-hand side F has a singularity of order $\gamma > \gamma_c$ at some point $a \in \Omega$, that is, $F(x) \simeq |x-a|^{-\gamma}$ in a neighbourhood of a , then at the same point the weak solution u has singularity of order which is larger than γ . The value of γ_c is optimal. For $p > 2$ we have the loss of regularity in the sense that if $F(x) = C|x|^m$ with $m > 0$, then $u(x) = u(0) + D|x|^\mu$ with $\mu < m$, provided $m > \gamma_c$. We show that the p -Laplace operator is not hypoelliptic for $p \in (1, \infty) \setminus \{1 + 1/n : n \in 2\mathbb{N} - 1\}$.

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REFERENCES

- [1] B. ACCIAIO, P. PUCCI, *Existence of radial solutions for quasilinear elliptic equations with singular nonlinearities*, Adv. Nonlinear Stud., **3**, 4 (2003), 511–539.
- [2] R. BORGHOL, L. VÉRON, *Boundary singularities of N -harmonic functions*, Comm. Partial Differential Equations, **32**, 4-6 (2007), 1001–1015.
- [3] P. DRÁBEK, A. KUFNER, F. NICOLOSI, *Quasilinear Elliptic Equations with Degenerations and Singularities*, de Gruyter Series in Nonlinear Analysis and Applications 5, Berlin - New York 1997.
- [4] K. FALCONER, *Fractal Geometry*, John Wiley & Sons, Chichester, 1999.
- [5] I. FONSECA, J. MALÝ, G. MINGIONE, *Scalar Minimizers with Fractal Singular Sets*, Arch. Rational Mech. Anal., **172** (2004), 295–307.
- [6] N. GHOUSSOUB, F. ROBERT, *Elliptic equations with critical growth and a large set of boundary singularities*, Trans. Amer. Math. Soc., **361** (2009), 4843–4870.
- [7] M. GRILLOT, *Prescribed singular submanifolds of some quasilinear elliptic equations*, Nonlinear Anal., **34**, 6 (1998), 839–856.
- [8] J. HEINONEN, T. KILPELÄINEN, O. MARTIO, *Nonlinear Potential Theory of Degenerate Elliptic Equations*, Oxford University Press, 1993.
- [9] L. HORVAT, D. ŽUBRINIĆ, *Maximally singular Sobolev functions*, J. Math. Anal. Appl., **304** (2005), 531–541.
- [10] G. MINGIONE, *Singularities of minima: a walk on the wild side of the calculus of variations*, J. Global Optim., **40**, 1-3 (2008), 209–223.
- [11] P. PUCCI, M. GARCÍA-HUIDOBRO, R. MANÁSEVICH, J. SERRIN, *Qualitative properties of ground states for singular elliptic equations with weights*, J. Differential Equations, **234**, 2 (2007), 507–543.
- [12] P. PUCCI, R. SERVADEI, *Regularity of weak solutions of homogeneous or inhomogeneous quasilinear elliptic equations*, Indiana Univ. Math. J., **57** (2008), 3329–3364.
- [13] J. SIMON, *Sur des équations aux dérivées partielles non linéaires*, Thèse, Paris, 1977.
- [14] F. DE THÉLIN, *Local regularity properties for the solutions of a nonlinear partial differential equation*, Nonlinear Analysis, **6**, 8 (1982), 839–844.

- [15] L. VÉRON, *Singularities of solutions of second order quasilinear equations*, Pitman Research Notes in Mathematics Series, **353**, Longman, Harlow, 1996.
- [16] L. VÉRON, *Singular p -harmonic functions and related quasilinear equations on manifolds*, Proceedings of the 2001 Luminy Conference on Quasilinear Elliptic and Parabolic Equations and Systems, 133–154 (electronic), Electron. J. Differ. Equ. Conf., 8, Southwest Texas State Univ., San Marcos, TX, 2002.
- [17] D. ŽUBRINIĆ, *Generating singularities of weak solutions of Laplace equations*, J. Math. Anal. Appl., **244** (2000), 10–16.
- [18] D. ŽUBRINIĆ, *Singular sets of Sobolev functions*, Comptes Rendus Acad. Sci. - Series I - Mathematics, **334** (2002), 539–544.
- [19] D. ŽUBRINIĆ, *Singular sets of Lebesgue integrable functions*, Chaos, Solitons & Fractals, **21** (2004), 1281–1287.
- [20] D. ŽUBRINIĆ, *Analysis of Minkowski contents of fractal sets and applications*, Real Anal. Exchange, **31**, 2 (2005/2006), 315–354.
- [21] D. ŽUBRINIĆ, *Maximally singular functions in Besov spaces*, Archiv Math., **87** (2006), 154–162.
- [22] D. ŽUBRINIĆ, *Generating singularities of weak solutions of p -Laplace equations on fractal sets*, Rocky M. Journal of Math., **39** (2009), 359–366.
- [23] D. ŽUBRINIĆ, *Hausdorff dimension of singular sets of Sobolev functions and applications*, in More Progress in Analysis, Proceedings of the 5th International ISAAC Congress, Eds. H.G.W. Begher and F. Nicolosi, World Scientific 2009, 793–802.
- [24] D. ŽUBRINIĆ, *Singular dimension of solution set of a class of p -Laplace equations*, Complex Var. Elliptic Equ., to appear.