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EXISTENCE AND NON–EXISTENCE OF ENTIRE POSITIVE SOLUTIONS FOR QUASILINEAR SYSTEMS WITH SINGULAR AND SUPER–LINEAR TERMS

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Abstract. We establish the results concerning existence and non-existence of entire positive solutions for the nonlinear elliptic systems

$$\begin{cases} -\Delta_p u = a(x)u^m + \lambda c(x)v^n, & x \in \mathbb{R}^N, \\ -\Delta_q v = b(x)v^l + \theta c(x)u^n, & x \in \mathbb{R}^N, \\ u, v > 0,\ x \in \mathbb{R}^N \text{ and } u \to 0, v \to 0 \text{ as } |x| \to \infty, \end{cases}$$

where $1 < p, q < N$ and $\lambda, \theta \geqslant 0$ are nonnegative parameters, $a, b, c : \mathbb{R}^N \to [0, \infty)$ are locally Hölder continuous functions not identically zero, and $-\infty < m < p-1, -\infty < l < q-1, \max\{p-1, q-1\} < n$. The main purpose of this paper is to extend the principal theorem of Xu and Yang in [23] which concerned single equation.

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