

ATTRACTIVITY AND POSITIVITY RESULTS FOR NONLINEAR FUNCTIONAL INTEGRAL EQUATIONS VIA MEASURE OF NONCOMPACTNESS

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Abstract. Using the techniques of some new measures of noncompactness we prove in this paper some existence theorems concerning the global attractivity and ultimate positivity of the solutions for a nonlinear functional integral equation. Our investigations are placed in the Banach space of real-valued functions defined, continuous and bounded on unbounded intervals together with the applications of a recent measure theoretic fixed point theorem of Dhage [7]. On one hand, our results generalize the attractivity results of Dhage [9] with a different method and the results of Banas and Rzepka [4] and Banas and Dhage [5] with similar method but under weaker conditions and on the other hand they are new to the literature as regards ultimate positivity of the solutions for nonlinear functional integral equations. A few realizations of the obtained results are also indicated.

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