

ON THE OSCILLATION OF SECOND ORDER LINEAR IMPULSIVE DIFFERENTIAL EQUATIONS

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Abstract. For the second order linear impulsive differential equation with oscillatory coefficient

$$\begin{cases} (r(t)x'(t))' + h(t)x(t) = 0, & t \neq t_k, t_k \geq t_0, k = 1, 2, \dots, \\ x(t_k^+) = a_k x(t_k), x'(t_k^+) = b_k x'(t_k), & k = 1, 2, \dots, \\ x(t_0^+) = x_0, x'(t_0^+) = x'_0, \end{cases} \quad (\text{E})$$

where h can be changed sign on $[t_0, \infty)$, by using the equivalence transformation, we establish an associated impulsive differential equation with damping and give oscillation criteria for the equation. As applications, we obtain oscillation theorems for Eq.(E). Moreover, an example is also given to illustrate the relevance of the results.

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