

POSITIVE SOLUTIONS OF SECOND ORDER MULTI-POINT BOUNDARY VALUE PROBLEMS WITH NON-HOMOGENEOUS BOUNDARY CONDITIONS

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Abstract. We are interested in the existence of positive solutions to multi-point boundary value problems for second order nonlinear differential equations with non-homogeneous boundary conditions. We show that results for the multi-point problems can be proved much in a similar way by methods available for the three point problem.

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