

ON A CLASS OF NONLOCAL ELLIPTIC PROBLEMS WITH CRITICAL GROWTH

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Abstract. This paper is concerned with the existence of positive solutions to the class of nonlocal boundary value problems of the Kirchhoff type

$$-\left[M\left(\int_{\Omega} |\nabla u|^2 dx\right)\right] \Delta u = \lambda f(x, u) + u^5 \text{ in } \Omega, u(x) > 0 \text{ in } \Omega \text{ and } u = 0 \text{ on } \partial\Omega,$$

where $\Omega \subset \mathbb{R}^N$, for $N=1,2$ and 3 , is a bounded smooth domain, M and f are continuous functions and λ is a positive parameter. Our approach is based on the variational method.

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