

ON THE SOLVABILITY OF NONLINEAR BOUNDARY VALUE PROBLEMS

JESÚS RODRÍGUEZ AND KRISTEN KOBYLUS ABERNATHY

Abstract. The focus of this paper is the study of nonlinear differential equations subject to general boundary conditions. We formulate sufficient conditions for the existence of solutions based on the dimension of the solution space of the corresponding linear, homogeneous equation and the "size" of the nonlinear terms. Our approach is based on the Lyapunov-Schmidt Procedure (Alternative Method).

Mathematics subject classification (2010): 34B15.

Keywords and phrases: boundary value problems, Schauder fixed point theorem, Lyapunov-Schmidt procedure, alternative method, projection.

REFERENCES

- [1] R.P. AGARWAL, S. DJEBALI, T. MOUSSAOUI AND O.G. MUSTAFA, *On the asymptotic integration of nonlinear differential equations*, J. Comput. Appl. Math., **2** (2007), 352–376.
- [2] S. BANCROFT, J.K. HALE AND D. SWEET, *Alternative problems for nonlinear functional equations*, J. Differential Equations, **4** (1968), 40–56.
- [3] L. CESARI, *Functional analysis and periodic solutions of nonlinear differential equations*, Contributions to Differential Equations, **1** (1963), 149–187.
- [4] L. CESARI, *Functional analysis and Galerkin's method*, Michigan Math. J., **11** (1964), 385–414.
- [5] S. CHOW AND J.K. HALE, *Methods of Bifurcation Theory*, Springer, Berlin, 1982.
- [6] D. L. ETHERIDGE AND J. RODRÍGUEZ, *Periodic solutions of nonlinear discrete-time systems*, Appl. Anal., **62** (1996), 119–137.
- [7] D. L. ETHERIDGE AND J. RODRÍGUEZ, *Scalar discrete nonlinear two-point boundary value problems*, J. Differ. Equations Appl., **4** (1998), 127–144.
- [8] A. HALANAY, *Solutions périodiques et presque-périodiques des systèmes d'équations aux différences finies*, Arch. Rational Mech. Anal., **12** (1963), 134–149.
- [9] J.K. HALE, *Applications of alternative problems*, Lecture Notes, **71-1**, Brown University, Providence, RI, 1971.
- [10] J.K. HALE, *Ordinary Differential Equations*, Robert E. Kreiger Publishing Company, Malabar, Florida, 1980.
- [11] D.C. LEWIS, *On the role of first integrals in the perturbation of periodic solutions*, Ann. of Math. (2), **63** (1956), 535–548.
- [12] J. RODRÍGUEZ, *An alternative method for boundary value problems with large nonlinearities*, J. Differential Equations, **43** (1982), 157–167.
- [13] J. RODRÍGUEZ, *Nonlinear differential equations under Stieltjes boundary conditions*, Nonlinear Anal., **7** (1983), 107–116.
- [14] J. RODRÍGUEZ, *On resonant discrete boundary value problems*, Appl. Anal., **19** (1985), 265–274.
- [15] J. RODRÍGUEZ, *Galerkin's method for ordinary differential equations subject to generalized nonlinear boundary conditions*, J. Differential Equations, **97** (1992), 112–126.
- [16] J. RODRÍGUEZ, *Nonlinear discrete Sturm-Liouville problems*, J. Math. Anal. Appl., **308** (2005), 380–391.

- [17] J. RODRÍGUEZ AND D. SWEET, *Projection methods for nonlinear boundary value problems*, J. Differential Equations, **58** (1985), 282–293.
- [18] J. RODRÍGUEZ AND P. TAYLOR, *Weakly nonlinear discrete multipoint boundary value problems*, J. Math. Anal. Appl., **329** (2007), 77–91.
- [19] J. RODRÍGUEZ AND P. TAYLOR, *Scalar discrete nonlinear multipoint boundary value problems*, J. Math. Anal. Appl., **330** (2007), 876–890.
- [20] J. RODRÍGUEZ AND P. TAYLOR, *Multipoint boundary value problems for nonlinear ordinary differential equations*, Nonlinear Anal., **68** (2008), 3465–3474
- [21] N. ROUCHE AND J. MAWHIN, *Ordinary Differential Equations*, Pitman, London, 1980.
- [22] W. SPEALMAN AND D. SWEET, *The alternative method for solutions in the kernel of a bounded linear functional*, J. Differential Equations, **37** (1980), 297–302.