Differential
Equations
& Applications
Volume 2, Number 4 (2010), 501–517

EXISTENCE OF POSITIVE SOLUTIONS OF A CLASS OF QUASILINEAR ELLIPTIC EQUATIONS ON $\mathbb{R}^N$

JING MO AND ZUODONG YANG

Abstract. In this paper we study the following problem: $-\Delta_p u + |u|^{p-2}u = k(x)f(u) + h(x)$, $x \in \mathbb{R}^N$, where $u \in W^{1,p}(\mathbb{R}^N)$, $u > 0$ in $\mathbb{R}^N$. Under appropriate assumptions on k, h and f, we prove that problem has at least two positive solutions.

Mathematics subject classification (2010): 35J25, 35J67, 35J20.

Keywords and phrases: nonhomogeneous quasilinear elliptic equation, Mountain Pass Theorem, Pohozaev type identity.

REFERENCES

[1] J. GARCIA-AZORERO AND I. PERAL, *Multiplicity of solutions for elliptic problems with critical exponent or with a nonsymmetric term*, Trans. Amer. Math. Soc., **323**, 2 (1991), 877–895.

[2] J. GARCIA-AZORERO AND I. PERAL, *Existence and non-uniqueness for the p-Laplacian: nonlinear eigenvalues*, Comm. Partial Differential Equations, **12** (1987), 1389–1430.

[3] J.F. BONDER AND J.D. ROSSI, *Existence results for the p-Laplacian with nonlinear boundary condition*, J. Math.Anal. Appl., **263**, 1 (2001), 195–223.

[4] I. EKELAND, *Convexity Methods in Hamiltonian Mechanics*, Springer, 1990.

[5] Z.M. GUO, *Existence and uniqueness of the positive radial solutions for a class of quasilinear elliptic equations*, Appl. Anal. **47**, 1 (1992), 173–190.

[6] Z.M. GUO AND Z.D. YANG, *Some uniqueness results for a class of quasilinear elliptic eigenvalue problems*, Acta Mathematica Sinica (new series), **14**, 2 (1998), 245–260.

[7] Z.M. GUO, *Some existence and multiplicity results for a class of quasilinear elliptic eigenvalue problems*, Nonlinear Anal., **18** (1992), 957–971.

[8] Z.M. GUO AND J.R.L. WEBB, *Uniqueness of positive solutions for quasilinear elliptic equations when a parameter is large*, Proc. Royal Soc. Edinburgh, **124**, A (1994), 189–198.

[9] Z.M. GUO, *On the number of positive solutions for quasilinear elliptic eigenvalue problems*, Nonlinear Anal., **27**, 2 (1996), 229–247.

[10] D.D. HAI, *Positive solutions of quasilinear boundary value problems,* J. Math.Anal. Appl., **217** (1998), 672–686.

[11] L. JEAN, *Two positive solutions for a class of nonhomogeneous elliptic equations*, Differential Integral Equations, **10** (1997), 609–624.

[12] L. JEAN, *On the existence of bounded Palais-Smale sequences and applications to a Landesman-Lazer-type set on* $\mathbf{R}^N$, Proc. Roy. Soc. Edinburgh Sect. A, **129** (1999), 787–809.

[13] Y.H. KIM, *A global bifurcation for nonlinear equations with nonhomogeneous part*, Nonlinear Analysis, doi:10.1016/j.na.2008.11.028.

[14] S.J. LI, S.P. WU AND H.S. ZHOU, *Solutions to semilunar elliptic problems with combined nonlinearities*, J. Differential Equations, **185** (2002), 200–224.

[15] Z.L. LIU AND Z.Q. WANG, *Existence of a positive solution of an elliptic equation on* $\mathbb{R}^N$, Proc. Roy. Soc. Edinburgh Sect. A, **134** (2004), 191–200.

[16] C. LIU, Z. WANG AND H.S. ZHOU, *symptotically linear Schrödinger equation with potential vanishing at infinity*, J. Differential Equations, **245** (2008), 201–222.

[17] C.A. STUART AND H.S. ZHOU, *Applying the mountain pass theorem to an asymptotically linear elliptic equation on* $\mathbb{R}^N$, Comm. Partial Differential Equations, **24** (1999), 1731–1758.

Paper DEA-02-30

[18] J.B. SU, Z.Q. WANG AND M. WILLEM, *Weighted Sobolev embedding with unbouded and decaying radial potentials*, J.Differential Equations, **238** (2007), 201–219.
[19] Z.P. WANG AND H.S. ZHOU, *Positive solutions for a nonhomogeneous elliptic equation on $\mathbb{R}^N$ without (AR) condition*, J. Math. Anal. Appl. **353** (2009), 470–479.
[20] B.J. XUAN AND Z.C. CHEN, *Solvability of singular quasilinear elliptic equation*, Chinese Ann. Math., **20A**, 1 (1999), 117–128.
[21] Z.D. YANG, *Existence of positive entire solutions for singular and non-singular quasi-linear elliptic equation*, J. Comput. Appl. Math., **197** (2006), 355–364.
[22] X.P. ZHU, *A perturbation result on positive entire solutions of a semilinear elliptic equation*, J. Differential Equations, **92** (1991), 163–178.
[23] X.P. ZHU AND H.S. ZHOU, *Existence of multiple positive solutions of inhomogeneous semilinear elliptic problems in unbounded domains*, Proc. Roy. Soc. Edinburgh Sect. A, **115** (1990), 301–318.