

## EXISTENCE OF POSITIVE SOLUTIONS OF A CLASS OF QUASILINEAR ELLIPTIC EQUATIONS ON $\mathbb{R}^N$

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*Abstract.* In this paper we study the following problem:  $-\Delta_p u + |u|^{p-2}u = k(x)f(u) + h(x)$ ,  $x \in \mathbb{R}^N$ , where  $u \in W^{1,p}(\mathbb{R}^N)$ ,  $u > 0$  in  $\mathbb{R}^N$ . Under appropriate assumptions on  $k$ ,  $h$  and  $f$ , we prove that problem has at least two positive solutions.

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