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A BREZIS–NIRENBERG TYPE THEOREM ON LOCAL MINIMIZERS FOR $p(x)$ –KIRCHHOFF DIRICHLET PROBLEMS AND APPLICATIONS

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Abstract. This paper deals with a class of $p(x)$-Kirchhoff Dirichlet problems possessing a variational structure which corresponds to the variational functional E defined on ${W_0}^{1,p(x)}(\Omega)$. We prove a Brezis-Nirenberg type theorem which asserts that every local minimizer of E in the $C^1(\overline{\Omega})$ topology is also a local minimizer of E in the $W_0^{1,p(x)}(\Omega)$ topology. Some applications of this theorem are given.

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