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ON AN EIGHTH ORDER OVERDETERMINED ELLIPTIC BOUNDARY VALUE PROBLEM

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Abstract. We consider the overdetermined boundary value problem for the 4-harmonic operator, $\Delta^4 = \Delta(\Delta^3)$, and show that if the solution of the problem exists, then the domain must be an open N-ball $(N \geqslant 2)$. As a consequence of overdetermined problems mean value results are obtained for harmonic, biharmonic, triharmonic and 4-harmonic functions.

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