

EXISTENCE OF POSITIVE SOLUTIONS FOR QUASILINEAR ELLIPTIC EQUATION ON RIEMANNIAN MANIFOLDS

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Abstract. Let (\mathcal{M}, g) be a smooth compact Riemannian manifold of dimension $n \geq 3$. We study the existence of positive weak solutions for the following quasilinear elliptic equation

$$-(\Delta_p)_g u + u^{p-1} = f(x, u, \nabla_g u) \quad \text{in } \mathcal{M},$$

where $(\Delta_p)_g u = \operatorname{div}_g(|\nabla u|_g^{p-2} \nabla u)$ is the p -Laplacian operator on Riemannian manifold (\mathcal{M}, g) with $1 < p < n$.

Mathematics subject classification (2010): 35J60, 53C21, 58J60.

Keywords and phrases: Riemannian manifold, quasilinear elliptic equation, a priori bound, existence of solution.

REFERENCES

- [1] C. AZIZIEH AND P. CLÉMENT, *A priori estimates and continuation methods for positive solutions of p -Laplace equations*, J. Differential Equations, **179** (2002), 213–245.
- [2] E. DI BENEDETTO, *$C^{1,\alpha}$ local regularity of weak solutions of degenerate elliptic equations*, Nonl. Anal., **7** (1983), 827–850.
- [3] H. BREZIS AND R. E. L. TURNER, *On a class of superlinear elliptic problems*, Comm. PDE., **2** (1977), 601–614.
- [4] V. BENCI, C. BONANNO, A. M. MICHELETTI, *On the multiplicity of solutions of a nonlinear elliptic problem on Riemannian manifolds*, J. Funct. Anal., **252** (2007), 464–489.
- [5] D. DE FIGUEIREDO, P. L. LIONS AND R. D. NUSSBAUM, *A priori estimates and existence of positive solutions of semilinear elliptic equations*, J. Math. Pures. Appl., **61** (1982), 41–63.
- [6] B. GIDAS AND J. SPRUCK, *A priori bounds for positive solutions of nonlinear elliptic equations*, Comm. PDE., **6** (1981), 883–901.
- [7] M. A. KRASNOSELSKII, *Fixed point of cone-compressing or cone-extending operators*, Soviet. Math. Dokl., **1** (1960), 1285–1288.
- [8] G. M. LIEBERMAN, *Boundary regularity for solutions of degenerate elliptic equations*, Nonlinear Anal., **12** (1988), 1203–1219.
- [9] A. G. LOSEV, Y. S. FEDORENKO, *Positive solutions of quasilinear elliptic inequalities on noncompact Riemannian manifolds*, Mathematical Notes, **81**, 6 (2007), 778–787.
- [10] S. PIGOLA, M. RIGOLI, A. G. SETTI, *A Liouville-type result for quasilinear elliptic equations on complete Riemannian manifolds*, Journal of Functional Analysis, **219** (2005), 400–432.
- [11] D. RUIZ, *A priori estimates and existence of positive solutions for strongly nonlinear problems*, J. Differential Equation, **199**, 1 (2004), 96–114.
- [12] J. SERRIN AND H. ZOU, *Cauchy-Liouville and universal boundedness theorems for quasilinear elliptic equations and inequalities*, Acta Mathematica, **189**, 1 (2002), 79–142.
- [13] P. TOLKSDORF, *Regularity for a more general class of quasilinear elliptic equations*, J. Differential Equation, **51** (1984), 126–150.
- [14] D. VISETTI, *Multiplicity of solutions of a zero mass nonlinear equation on a Riemannian manifold*, J. Differential Equations, **245** (2008), 2397–2439.
- [15] HENG HUI ZOU, *A priori estimates and existence for quasi-linear elliptic equations*, Calc. Var PDE., **33** (2008), 417–437.

