

ON THE DIRICHLET PROBLEM OF LANDAU–LIFSHITZ–MAXWELL EQUATIONS

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Abstract. We prove the existence and uniqueness of non-trivial stable solutions to Landau-Lifshitz-Maxwell equations with Dirichlet boundary condition for large anisotropies and small non-simply connected domains.

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