STABILITY OF FUNCTIONAL DIFFERENTIAL EQUATIONS WITH OSCILLATING COEFFICIENTS AND DISTRIBUTED DELAYS

MICHAEL I. GIL’

Abstract. We consider the scalar equation
\[ \dot{x}(t) + \sum_{j=1}^{m} a_j(t) \int_{0}^{h} x(t-s) r_j(s) \, ds = 0 \quad (h = \text{const} > 0, \dot{x} = dx/dt), \]
where \( r_j(s) \) are nondecreasing functions. Besides, we do not require that \( a_j(t) \) are positive for all \( t \geq 0 \). So the function
\[ z + \sum_{j=1}^{m} a_j(t) \int_{0}^{h} e^{-zs} r_j(s) \, ds \]
can have zeros in the right-hand plane for some \( t \geq 0 \). It is proved that the considered equation is exponentially stable, provided \( a_j(t) = b_j + c_j(t) \), where \( b_j \) are positive constants, such that all the zeros of the function \( z + \sum_{j=1}^{m} b_j \int_{0}^{h} e^{-zs} r_j(s) \, ds \) are in the open left-hand plane, and the integrals \( \int_{0}^{h} c_j(s) \, ds \) \((j = 1, \ldots, m)\) are sufficiently small for all \( t > 0 \).


Keywords and phrases: functional differential equation, linear equation, exponential stability.

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