

IMPLICIT DIFFERENCE INEQUALITIES CORRESPONDING TO PARABOLIC FUNCTIONAL DIFFERENTIAL EQUATIONS

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Abstract. We give theorems on implicit difference inequalities generated by initial-boundary value problems for parabolic functional differential equations. We apply this result for the investigation of the stability of difference schemes. Classical solutions of mixed problems are approximated in the paper by solutions of suitable implicit difference methods. The proofs of the convergence of difference methods are based on a comparison technique and the results on difference functional inequalities are used. Numerical examples are presented.

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