

## EXISTENCE OF POSITIVE SOLUTIONS FOR QUASILINEAR ELLIPTIC SYSTEMS WITH SOBOLEV CRITICAL EXPONENTS

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*Abstract.* In this paper, we consider the existence of positive solutions to the following problem

$$\begin{cases} -\operatorname{div}(|\nabla u|^{p-2}\nabla u) = \frac{\partial F}{\partial u}(u, v) + \varepsilon^{p-1}g(x) & \text{in } \Omega, \\ -\operatorname{div}(|\nabla v|^{q-2}\nabla v) = \frac{\partial F}{\partial v}(u, v) + \varepsilon^{q-1}h(x) & \text{in } \Omega, \\ u, v > 0 & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases}$$

where  $\Omega$  is a bounded smooth domain in  $\mathbb{R}^N$ ;  $F \in C^1((\mathbb{R}^+)^2, \mathbb{R}^+)$  is positively homogeneous of degree  $\mu$ ;  $g, h \in C^1(\overline{\Omega}) \setminus \{0\}$ ; and  $\varepsilon$  is a positive parameter. Using sub-supersolution method and comparison principle, we prove the existence of positive solutions for the above problem.

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