# NONLINEAR DEGENERATE DIFFUSION
# PROBLEMS WITH A SINGULARITY

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*Abstract.* We consider a class of degenerate nonlinear diffusion problems with a singularity in a finite value $M > 0$ of the unknown $v$. For such problems, we introduce a notion of renormalized entropy solution which (under a particular "growth" assumptions on the diffusion term) can reach the value $M$. We prove the existence of such a solution for the stationary equation with $L^1$ data.

*Mathematics subject classification* (2010): 35J60, 35K35, 35K55.

*Keywords and phrases*: degenerate, homogenous boundary conditions, renormalized entropy solution, continuous flux, integrable data.

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