EXISTENCE OF POSITIVE ENTIRE SOLUTIONS OF A SEMILINEAR
p–LAPLACIAN PROBLEM WITH A GRADIENT TERM

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Abstract. In this paper, we study a semilinear p-Laplacian problem

$$-\Delta_p u + h(x)|\nabla u|^q = b(x)g(u),\ u > 0,\ x \in \mathbb{R}^N,\ \lim_{|x|\to\infty} u(x) = 0,$$

where $q \in (p-1, p]$, b, $h \in C^{\alpha}_{loc}(\mathbb{R}^N)$ for some $\alpha \in (0,1)$, $h(x) \geqslant 0$, $b(x) > 0, \forall x \in \mathbb{R}^N$, and $g \in C^1((0,\infty),(0,\infty))$ which may be singular at 0. Using a sub-supersolution argument and a perturbed argument, we obtain the existence of entire solutions to the problem. No monotonicity condition is imposed on the functions $g(s)$ and $\frac{g(s)}{s^{p-1}}$.

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