

EXISTENCE OF POSITIVE SOLUTIONS TO A QUASILINEAR ELLIPTIC SINGULAR NEUMANN PROBLEM

QING MIAO AND ZUODONG YANG

Abstract. We show the existence of positive solution for the following singular Neumann problem

$$\begin{cases} -\operatorname{div}(|\nabla u|^{m-2}\nabla u) + \frac{a(x)}{u^\beta} = \lambda h(x)u^p & \text{in } B_R, \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial B_R, \end{cases}$$

where $R > 0, \lambda > 0$ is a positive parameter, $\beta > 0, p \in [0, m-1)$. By means of double perturbation argument and variational methods, we obtain a positive solution $u \in C^1(\overline{B}_R \setminus \{0\}) \cap C(\overline{B}_R)$.

Mathematics subject classification (2010): 35J25, 35J65.

Keywords and phrases: quasilinear elliptic equation, sub-supersolution, variational method, Neumann conditions.

REFERENCES

- [1] C. O. ALVES, M. MONTENEGRO, *Positive solutions to a singular Neumann problem*, J. Math. Anal. Appl., **352** (2009), 112–119.
- [2] G. ASTRITA, G. MARRUCCI, *Principles of non-Newtonian fluid mechanics*, McGraw-Hill, 1974.
- [3] H. BREZIS, L. NIRENBERG, *Positive solutions of nonlinear elliptic equations involving critical Sobolev exponents*, Comm. Pure Appl. Math., **36** (1983), 437–477.
- [4] F. CIRSTEA, V. RADULESCU, *Existence and uniqueness of positive solutions to a semilinear elliptic problem in \mathbb{R}^N* , J. Math. Anal. Appl., **229** (1999), 417–425.
- [5] M. G. CRANDALL, P. H. RABINOWITZ, L. TARTAR, *On a Dirichlet problem with singular nonlinearity*, Comm. Partial Differential Equations, **2** (1997), 193–222.
- [6] J. I. DIAZ, J. M. MOREL, L. OSWALD, *An elliptic equation with singular nonlinearity*, Comm. Partial Differential Equations, **12**, 12 (1987), 1333–1344.
- [7] A. EDELSON, *Entire solutions of singular elliptic equations*, J. Math. Anal. Appl., **139** (1989), 523–532.
- [8] Z. M. GUO, *Some existence and multiplicity results for a class of quasilinear elliptic eigenvalue problems*, Nonlinear Anal., **18** (1992), 957–971.
- [9] Z. M. GUO, *Existence and uniqueness of positive radial solutions for a class of quasilinear elliptic equations*, Appl. Anal., **47** (1992), 173–190.
- [10] Z. M. GUO, *Boundary value problems of a class of quasilinear ordinary differential equations*, Differential Integral Equations, **6**, 3 (1993), 705–719.
- [11] Z. M. GUO, *On the number of positive solutions for quasilinear elliptic eigenvalue problems*, Nonlinear Anal., **27**, 2 (1996), 229–247.
- [12] Z. M. GUO, J. R. L. WEBB, *Uniqueness of positive solutions for quasilinear elliptic equations when a parameter is large*, Proc. Roy. Soc. Edinburgh. Sect. A, **124** (1994), 189–198.
- [13] Z. M. GUO, J. R. L. WEBB, *Large and small solutions of a class of quasilinear elliptic eigenvalue problems*, J. Differential Equations, **180** (2002), 1–50.
- [14] A. S. KALASHNIKOV, *On a nonlinear equation appearing in the theory of non-stationary filtration*, Trudy Sem. Petrovsk., **4** (1978), 137–146.
- [15] T. KUSANO, C. A. SWANSON, *Entire positive solutions of singular semilinear elliptic equations*, Japan J. Math., **11** (1985), 145–155.

- [16] A. C. LAZER, P. J. MCKENNA, *On a singular nonlinear elliptic boundary value problem*, Proc. Amer. Math. Soc., **111**, 3 (1991), 721–730.
- [17] A. V. LAIR, A. W. SHAKER, *Entire solution of a singular semilinear elliptic problem*, J. Math. Anal. Appl., **200** (1996), 498–505.
- [18] Q. S. LU, Z. D. YANG, E. H. TWIZELL, *Existence of entire explosive solutions of quasilinear elliptic equations*, Appl. Math. Comput., **148** (2004), 359–372.
- [19] L. K. MARTINSON, K. B. PAVLOV, *Unsteady shear flows of a conducting fluid with a rheological power law*, Magnitnaya Gidrodinamika, **2** (1971), 50–58.
- [20] Q. MIAO, Z. D. YANG, *Bounded Positive Entire Solutions of Singular p -Laplacian Equations*, Nonlinear Anal., **69** (2008), 3749–3760.
- [21] M. DEL PINO, G. HERNANDEZ, *Solvability of the Neumann problem in a ball for $-\Delta u + u^{-v} = h(|x|)$, $v > 1$* , J. Differential Equations, **124** (1996), 108–131.
- [22] O. S. DE QUEIROZ, *A Neumann problem with logarithmic nonlinearity in a ball*, Nonlinear Anal., **70** (2009), 1656–1662.
- [23] B. J. XUAN, Z. C. CHEN, *Solvability of singular quasilinear elliptic equation*, Chinese Ann. Math. Ser. A, **20**, 1 (1999), 117–128.
- [24] Z. D. YANG, *Non-existence of positive entire solutions for elliptic inequalities of p -laplacian*, Appl. Math. J. Chinese Univ. Ser. B, **12**, 4 (1997), 399–410.
- [25] Z. D. YANG, *Existence of entire explosive positive radial solutions for a class of quasilinear elliptic systems*, J. Math. Anal. Appl., **288**, 2 (2003), 768–783.
- [26] Z. D. YANG, *Existence of positive entire solutions for singular and non-singular quasi-linear elliptic equation*, J. Comput. Appl. Math., **197**, 2 (2006), 355–364.
- [27] Z. D. YANG, Q. S. LU, *Blow-up estimates for a quasilinear reaction-diffusion system*, Math. Methods Appl. Sci., **26** (2003), 1005–1023.
- [28] Z. D. YANG, Q. S. LU, *Nonexistence of positive solutions to a quasilinear elliptic system and blow-up estimates for a quasilinear reaction-diffusion system*, J. Comput. Appl. Math., **50** (2003), 37–56.