

LONG TIME ASYMPTOTICS OF SUB-THRESHOLD SOLUTIONS OF A SEMILINEAR CAUCHY PROBLEM

JARI TASKINEN

Abstract. We show that the solution of the semilinear heat equation $u_t = u_{xx} + u^p$ (with $x \in \mathbb{R}$, $p > 3$, and nonnegative Cauchy data) behaves for large t like the solution of the corresponding linear problem plus a small correction of order $t^{-1/2-c}$, where $c := 1/2$, if $p \geq 4$, and $c = (p-3)/2$, if $3 < p < 4$. The result is known in special cases like small initial data. We prove it here for positive sub-threshold initial data satisfying some assumptions. Part of our results are contained in the recent work [10], but the motivation of this paper is to provide a new method leading to somewhat more general space-time estimates.

Mathematics subject classification (2010): 35K57, 35K05, 35K91.

Keywords and phrases: semilinear heat equation, reaction-diffusion equation, Cauchy problem, asymptotic behaviour, small data.

REFERENCES

- [1] J. BRICMONT, A. KUPIAINEN, AND G. LIN, *Renormalization group and asymptotics of solutions of nonlinear parabolic equations*, *Comm. Pure Appl. Math.*, **47** (1994), 839–922.
- [2] M. FILA, J. KING, M. WINKLER, AND E. YANAGIDA, *Linear behaviour of solutions of a superlinear heat equation*, *J. Math. Anal. Appl.*, **340** (2008), 401–409.
- [3] H. FUJITA, *On the blowing up of the solutions of the Cauchy problem for $u_t = \Delta u + u^{1+\alpha}$* , *J. Fac. Sci. Univ. Tokyo Sect I*, **13** (1966), 109–124.
- [4] V. GALAKTIONOV, AND J.L. VÁZQUEZ, *A stability technique for evolution partial differential equations: a dynamical systems approach*, Birkhäuser, Boston, 2004.
- [5] Y. GIGA, *A bound for global solutions of semilinear heat equations*, *Comm. Math. Phys.*, **103** (1986), 415–421.
- [6] A. GMIRA, AND L. VERON, *Large time behavior of the solutions of a semilinear parabolic equation in \mathbb{R}^N* , *J. Differential Equations*, **53** (1984), 258–276.
- [7] C. GUI, W.-M. NI, AND X. WANG, *On the stability and instability of positive steady states of a semilinear heat equation in \mathbb{R}^N* , *Comm. Pure Appl. Math.*, **45** (1992), 1153–1181.
- [8] C. GUI, W.-M. NI, AND X. WANG, *Further study of a nonlinear heat equation*, *J. Differential Equations*, **169** (2001), 588–613.
- [9] L. A. HERRAIZ, *Asymptotic behavior of solutions of some semilinear parabolic problems*, *Ann. Inst. H. Poincaré Anal. Non Linéaire*, **16** (1999), 49–105.
- [10] K. ISHIGE, M. ISHIWATA, AND T. KAWAKAMI, *The decay of the solutions for the heat equation with a potential*, *Indiana Univ. Math. J.*, **58**, 6 (2009), 2673–2707.
- [11] K. ISHIGE, AND T. KAWAKAMI, *Asymptotic behavior of solutions for some semilinear heat equations in \mathbb{R}^N* , *Commun. Pure Appl. Anal.*, **8**, 4 (2009), 1351–1371.
- [12] S. KAMIN, AND L. A. PELETIER, *Large time behaviour of solutions of the heat equation with absorption*, *Ann. Scu. Norm. Sup. Pisa Cl. Sci.*, **12** (1985), 393–408.
- [13] O. KAVIAN, *Remarks on the large time behaviour of a nonlinear diffusion equation*, *Ann. Inst. H. Poincaré, Anal. Non Linéaire*, **4** (1987), 423–452.
- [14] T. KAWANAGO, *Asymptotic behavior of solutions of a semilinear heat equation with subcritical nonlinearity*, *Ann. Inst. H. Poincaré, Anal. Non Linéaire*, **13** (1996), 1–15.

- [15] K. KOBAYASHI, T. SIRAO, AND H. TANAKA, *On the growing up problem for semi-linear heat equations*, J. Math. Soc. Japan, **29** (1977), 407–424.
- [16] T. LEE, AND W.-M. NI, *Global existence, large time behaviour and life span of solutions of a semi-linear parabolic Cauchy problem*, Trans. Amer. Math. Soc., **333**, 1 (1992), 365–378.
- [17] T.-T. LI, AND Y.-M. CHEN, *Global classical solutions for nonlinear evolution equations*, Pitman Monographs and Surveys in Pure and Applied Mathematics, 45. Longman Scientific & Technical, Harlow; copublished in the United States with John Wiley & Sons, Inc., New York, 1992.
- [18] P. POLÁČEK, AND P. QUITTNER, *Asymptotic behavior of threshold and sub-threshold solutions of a semilinear heat equation*, Asymptot. Anal., **57**, 3–4 (2008), 125–141.
- [19] P. POLÁČEK, AND E. YANAGIDA, *On bounded and unbounded global solutions of a supercritical semilinear heat equation*, Math. Ann., **327** (2003), 745–771.
- [20] P. POLÁČEK, AND E. YANAGIDA, *A Liouville property and quasiconvergence for a semilinear heat equation*, J. Differential Equations, **208** (2005), 194–214.
- [21] P. QUITTNER, *The decay of global solutions of a semilinear heat equation*, Discrete Contin. Dyn. Syst., **21**, 1 (2008), 307–318.
- [22] P. QUITTNER, personal communication.
- [23] P. QUITTNER, AND PH. SOUPLÉ, *Superlinear parabolic problems. Blow-up, global existence and steady states*, Birkhäuser Advanced Texts, Basel, 2007.
- [24] A. SAMARSKII, V. GALAKTIONOV, S. KURDYUMOV, AND A. MIKHAILOV, *Blow-up in quasilinear parabolic equations*, de Gruyter Expositions in Mathematics **19**, Walter de Gruyter & Co., Berlin, 1995.
- [25] J. TASKINEN, *Asymptotical behaviour of a class of semilinear diffusion equations*, J. Evol. Equ., **7** (2007), 429–447.
- [26] H. ZHAO, *Large time decay estimates of solutions of nonlinear parabolic equations*, Discrete Contin. Dyn. Syst., **8**, 1 (2002), 69–144.
- [27] S.-M. ZHENG, *Nonlinear Parabolic Equations and Hyperbolic-Parabolic Coupled Systems*, Pitman Monographs and Surveys in Pure and Applied Mathematics, **76**, Longman Scientific & Technical, Harlow, 1995.