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LONG TIME ASYMPTOTICS OF SUB–THRESHOLD SOLUTIONS OF A SEMILINEAR CAUCHY PROBLEM

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Abstract. We show that the solution of the semilinear heat equation $u_t = u_{xx} + u^p$ (with $x \in \mathbb{R}$, $p > 3$, and nonnegative Cauchy data) behaves for large t like the solution of the corresponding linear problem plus a small correction of order $t^{-1/2-c}$, where $c := 1/2$, if $p \geqslant 4$, and $c = (p-3)/2$, if $3 < p < 4$. The result is known in special cases like small initial data. We prove it here for positive sub-threshold initial data satisfying some assumptions. Part of our results are contained in the recent work [10], but the motivation of this paper is to provide a new method leading to somewhat more general space-time estimates.

Mathematics subject classification (2010): 35K57, 35K05, 35K91.

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