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SOME NEW OSCILLATION CRITERIA FOR HIGHER–ORDER QUASI–LINEAR NEUTRAL DELAY DIFFERENTIAL EQUATIONS

YANYUN QIAN AND RUN XU

Abstract. Oscillation criteria for the higher order quasi-linear neutral delay differential equations of the form

$$[r(t)\psi(u(t))|z^{(n-1)}(t)|^{\alpha-1}z^{(n-1)}(t)]' + \sum_{i=1}^{m} q_i(t)f_i(|u(\tau_i(t))|^{\alpha_i-1}u(\tau_i(t))) = 0,$$

$t \geqslant t_0$, $z(t) = u(t) + p(t)u(t-\sigma), \alpha > 0, \alpha_i > 0$ $(i = 1,2,3,...,m)$, are established under the condition:

$$\int_{t_0}^{\infty} r^{-\frac{1}{\alpha}}(s)ds = \infty \text{ or } \int_{t_0}^{\infty} r^{-\frac{1}{\alpha}}(s)ds < \infty \text{ respectively,}$$

where n is even. The obtained results improve and extend some known results in literature.

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