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EXISTENCE OF SOLUTIONS TO BOUNDARY VALUE PROBLEMS AT FULL RESONANCE

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Abstract. The focus of this paper is the study of nonlinear differential equations of the form

$$\dot{x}_i(t) = a_i(t)x_i(t) + f_i(\varepsilon, t, x_1(t), \cdots, x_n(t)), \;\; i = 1, 2, \cdots, n,$$

subject to two-point boundary conditions

$$b_i x_i(0) + d_i x_i(1) = 0, \;\; i = 1, 2, \cdots, n.$$

We formulate sufficient conditions for the existence of solutions based on the dimension of the solution space of the corresponding linear, homogeneous equation and the properties of the nonlinear term when $\varepsilon = 0$. We focus on the case when the solution space of the corresponding linear, homogeneous equation is n-dimensional; that is, when the system is at full resonance. The argument we use relies on the Lyapunov-Schmidt procedure and the Schauder fixed point theorem.

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