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POSITIVE SOLUTIONS TO A TWO POINT SINGULAR BOUNDARY VALUE PROBLEM

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Abstract. We employ fixed point index theory to establish existence results for positive solutions to the singular boundary value problem

$$\begin{cases} -(au')'(t) = b(t)f(t,u(t)), \quad t \in (0,1), \\ u'(0) = u(1) = 0, \end{cases}$$

where $a \in C^1((0,1),(0,\infty))$, $1/a$ is integrable on any compact subset of $(0,1]$, $b \in C((0,1),[0,+\infty))$ does not vanish identically and is integrable on any compact subset of $[0,1)$, and $f : [0,1] \times \mathbb{R}^+ \to \mathbb{R}^+$ is continuous with $f(t,u) > 0$ for all $(t,u) \in [0,1] \times (0,\infty)$. As applications, existence and nonexistence criteria for positive radial solutions to some elliptic equations are deduced.

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