Differential
Equations
& Applications
Volume 3, Number 3 (2011), 375–384

PERIODIC SOLUTIONS OF LIÉNARD EQUATION WITH ONE OR TWO WEAK SINGULARITIES

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Abstract. In this paper we study the existence and asymptotic stability of periodic solutions of the differential equation

$$\ddot{x}+f(x)\dot{x}+g(x)=h(t),$$

where $h(t)$ is T-periodic, $f(x)$ is positive and $g(x)$ is strictly monotonically increasing and has one or two weak singularities. The method of proof relies on the construction of a positively invariant region of the flux.

Mathematics subject classification (2010): 34C25.
Keywords and phrases: periodic solution, singularity, asymptotic stability.

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Paper DEA-03-23

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