

ON THE ASYMPTOTIC BEHAVIOURS OF SOLUTIONS OF THIRD ORDER NON-LINEAR AUTONOMOUS DIFFERENTIAL EQUATION GOVERNING THE MHD FLOW

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Abstract. This paper deals with the asymptotic behaviour as $t \rightarrow \infty$ of the solutions for a steady laminar incompressible boundary layer equations governing the MHD flow near the forward stagnation point of two-dimensional and axisymmetric bodies. The asymptotic behaviour of the solutions is based on the method of asymptotic integration of second order linear differential equations. The results pertaining to the asymptotic behaviour of the solutions are also expressed in the form of Theorems 4.1 and 4.2.

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