SMALL DATA SCATTERING FOR A SYSTEM
OF NONLINEAR SCHRÖDINGER EQUATIONS

NAKAO HAYASHI, CHUNHUA LI AND TOHRU OZAWA

Abstract. We study the scattering theory for a system of nonlinear Schrödinger equations in space dimension $n \geqslant 3$. In the case $n \geqslant 4$, existence of the scattering operator is proved in small data setting in the Sobolev space $H^{n/2-2}$. In the case $n = 3$, a similar result is proved in the weighted L^2 space $\langle x \rangle^{-1/2} L^2 = \mathscr{F}(H^{-1/2})$ under the mass resonance condition.

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